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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. FIRST SEMESTER EXAMINATION, DECEMBER 2015

FIRST YEAR [BATCH 2015-18]

Date : 19/12/2015 **STATISTICS (General)**

Time : 11 am – 1 pm Paper : I Full Marks : 50

[Use a separate Answer Book for each group]

Group – A

1. Answer **any three** questions : (3x5)

   a) Discuss briefly the various methods of graphical representation of frequency distribution of different types.

   b) If a variable x takes the values 1, 2, …. k with $F_1, F_2, \ldots F_k$ as the corresponding less than type cumulative frequencies, then show that $\bar{x} = (k+1) - \frac{1}{n}\sum_{i=1}^{k} F_i$ where n is the total frequency.

   c) Find mean deviation about mean and the standard deviation of the following series of values: $a, a+b, a+2b, \ldots\ldots., a+2nb$ $(b > 0)$.

   d) Let σ and R be, respectively, the s.d. and range of a set of n values of x, show that $\frac{R^2}{2n} \le \sigma^2 \le \frac{R^2}{4}$.

   e) The marks obtained by 40 students are grouped in a frequency table in class intervals of 10 marks each. The mean and variance obtained from this distribution are found to be 40 and 49 respectively. It is found, on verification, that 2 observations belonging to the class interval (21 – 30) are included in the class interval (31 – 40) by mistake. Find the mean and variance after correcting the error.

   f) Suppose the mean and sd of two groups of n_1 and n_2 observations are respectively $\bar{x}_1, \sigma_1$ and $\bar{x}_2, \sigma_2$.

      Show that the variance (σ^2) of the combined group is given by

      $$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2.$$

Answer either **Question No. 2** or **Question No. 3**: (1×10)

2. a) Prove that r_{xy}, the correlation coefficient between x and y satisfies $-1 \le r_{xy} \le 1$. (5)

   b) Show how the correlation coefficient between x and y changes for change of origin and scale. (5)

3. a) Derive the Least Square linear regression equation of y on x. (5)

   b) The equations of two regression lines are given as $5x+7y = 17$ and $3x + y = 7$. (5)

      Find (i) a.m. of x and y

      (ii) correlation coefficient between x and y

      (iii) ratio of variances of x and y

Group – B

Answer **any three** questions: (3x5)

4. a) Explain the Axiomatic theory of probability.

   b) Show that mutual independence of events implies their pair wise independence but the converse may not be true. (3+2)

5. A bag contains 50 tickets numbered 1, 2, 3, ….. 50 of which 5 are drawn at random and arranged in ascending order of their numbers $x_1 < x_2 < \ldots\ldots < x_5$. What is the probability that $x_3 = 30$? (5)

6. Each of three urn contains 'α' white and 'β' black balls. One ball is transferred from the 1st to the 2nd urn at random. Then a ball is transferred from 2nd to the 3rd urn at random. Then a ball is drawn from 3rd urn. Find the probability that it is white. (5)

7. There are 3 coins, identical in appearance, one of which is ideal and the other two biased with probabilities $\frac{1}{3}$ & $\frac{2}{3}$ respectively for a head. One coin is taken at random and tossed twice. If a head appears both times, then what is the probability that the ideal coin was chosen? (5)

8. In 10 independent throws of a defective die the probability that an even number will appear 5 times is twice the probability that an even number will appear 4 times. Find the probability that an even number will not appear at all in 10 independent throws of the die. (5)

9. Show that for a symmetric probability distribution, all odd ordered central moments vanish. (5)

Answer either **Question No. 10** or **Question No. 11**: (1×10)

10. a) The distribution function of a random variable X is defined as follows:

$$\begin{aligned} F_X(x) &= A; \ -\infty < x < -1 \\ &= B; \ -1 \le x < 0 \\ &= C; \ \ 0 \le x < 2 \\ &= D; \ \ 2 \le x < \infty \end{aligned}$$

Where A, B, C, D are constants. Determine the values of A, B, C, D, it being given that $P[X=0]=\dfrac{1}{6}$ & $P[X>1]=\dfrac{2}{3}$.

   b) For a random variable X, show that the function $f(x)$ given by

$$\begin{aligned} f(x) &= \ \ x; \quad 0<x<1 \\ &= (k-x); \ 1<x<2 \\ &= \ \ 0; \quad \text{elsewhere} \end{aligned}$$

is a p.d.f. for a suitable value of the constant k. Calculate the probability that the random variable lies between $\frac{1}{2}$ and $\frac{3}{2}$. (5+5)

11. a) If the possible values of the random variable X are 0, 1, 2, ……,

show $E(X)=\sum_{x=0}^{\infty}[1-F(x)]$.

   b) n balls are distributed at random among m cells. Find the expected number of cells that remain empty.

   c) The radius X of a circle is a random variable with p.d.f.

$$\begin{aligned} f(x) &= 1 \ \text{ for } \ 1<x<2 \\ &= 0 \ \text{ elsewhere} \end{aligned}$$

. Find the mean & variance of the area of the circle. (3+3+4)

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